Prime Factors Problems

Time: 1 hour and 25 minutes

Score: _____/84

Surname: .....

Other names: .....

Mark Scheme and revision available:
www.addvancemaths.com/revision/primefactors

Instructions

- Use black ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
- If blank paper is used, write down the question's number
- You must show all your working out.

Information

- The marks for each question are shown in brackets.
- Blank paper is provided at the end if extra space is needed.
- The questions are arranged in order of increasing difficulty.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Prime Factors Problems

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1) Prime Factorise these numbers:

$12 = 2^2 \times 3$ $15 = 3 \times 5$ $18 = 2 \times 3^2$

$19 = 19$ $40 = 2^3 \times 5$ $100 = 2^2 \times 5^2$

$190 = 2 \times 5 \times 19$ $400 = 2^4 \times 5^2$ $1000 = 2^3 \times 5^3$

$125 = 5^3$ $250 = 2 \times 5^3$ $84 = 2^2 \times 3 \times 7$

(12)

2) Find the highest common factor of the following pairs:

↖ Choose the smallest power from the primes that they share, then multiply.

a) 12 and 15 $2^2 \times 3$ 3×5 3 **(2)**

b) 40 and 12 $2^3 \times 5$ $2^2 \times 3$ 4 **(2)**

c) 1000 and 700 $2^3 \times 5^3$ $2^2 \times 5^2 \times 7$ $2^2 \times 5^2 = 100$ **(2)**

d) 36 and 48 $2^2 \times 3^2$ $2^4 \times 3$ $2^2 \times 3 = 12$ **(2)**

e) 700 and 280 $2^2 \times 5^2 \times 7$ $2^3 \times 5 \times 7$ $2^2 \times 5 \times 7 = 140$ **(2)**

f) 640 and 480 $2^7 \times 5$ $2^5 \times 3 \times 5$ $2^5 \times 5 = 160$ **(2)**

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3) Find the lowest common multiple of the following pairs on numbers:

↑ Choose the highest power of every prime in either prime factorisation, then multiply.

a) 12 and 15

$2^2 \times 3$ 3×5

$2^2 \times 3 \times 5 = 60$ **(2)**

b) 40 and 13

$2^3 \times 5$ 13

$2^3 \times 5 \times 13 = 520$ **(2)**

c) 1000 and 700

$2^3 \times 5^3$ $2^2 \times 5^2 \times 7$

$2^3 \times 5^3 \times 7 = 7000$ **(2)**

d) 36 and 48

$2^2 \times 3^2$ $2^4 \times 3$

$2^4 \times 3^2 = 144$ **(2)**

e) 700 and 280

$2^2 \times 5^2 \times 7$ $2^3 \times 5 \times 7$

$2^3 \times 5^2 \times 7 = 1400$ **(2)**

f) 640 and 480

$2^7 \times 5$ $2^4 \times 3$

$2^7 \times 3 \times 5 = 1920$ **(2)**

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4) Find the lowest common multiple of the following sets of numbers:

a) 12, 15 and 18

$2^2 \times 3$ 3×5 2×3^2

$2^2 \times 5 \times 3^2 = 180$ **(3)**

b) 400, 800 and 120

$2^4 \times 5^2$ $2^5 \times 5^2$ $2^3 \times 3 \times 5$

$2^5 \times 3 \times 5^2 = 2400$ **(3)**

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5) Consider $A = 2^3 \times 5^x \times 7^8$

Write the following as products of their prime factors:

a) $15A =$

$2^3 \times 3 \times 5^{x+1} \times 7^8$ **(2)**

b) $4A =$

$2^5 \times 5^x \times 7^8$ **(2)**

c) $A^3 =$

$2^9 \times 5^{3x} \times 7^{24}$ **(2)**

d) $\frac{3A}{7} =$

$2^3 \times 3 \times 5^x \times 7^7$ **(2)**

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6) Consider $A = 2^m \times 3^n \times 5^2 \times 7$ and $B = 2 \times 3^n \times 5^4 \times 11$ where m and n are integers larger than 2.

Write the following as products of their prime factors:

a) $AB =$

$2^{m+1} \times 3^{2n} \times 5^6 \times 7 \times 11$ **(2)**

b) $2AB^2 =$

$2^{m+3} \times 3^{3n} \times 5^{10} \times 7 \times 11^2$ **(3)**

c) The highest common factor of A and B

$2 \times 3^n \times 5^2$ **(2)**

d) The lowest common multiple of A and B

$2^m \times 3^n \times 5^4 \times 7 \times 11$ **(2)**

Prime Factors Problems

7) Consider $A = 2^{10} \times 3^{20} \times 5^{30}$

Write the following as products of their prime factors:

a) $10A =$

$2^{11} \times 3^{20} \times 5^{31}$ **(2)**

b) $\sqrt{A} =$

$2^{5} \times 3^{10} \times 5^{15}$ **(2)**

c) $\sqrt{81A}$

$2^{5} \times 3^{12} \times 5^{15}$ **(2)**

d) $\sqrt[5]{A}$

$2^{2} \times 3^{4} \times 5^{6}$ **(2)**

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8a) Write $2^{12} \times 3^3 \times 5^{11}$ in standard form.
Show your working.

$2 \times 3^3 \times 2^{11} \times 5^{11}$

$54 \times 10^{11} = 5.4 \times 10^{12}$

5.4×10^{12} **(3)**

b) The number of radioactive atoms in a sample of in a sample is 7.5×10^{28}.

75×10^{27}

$3 \times 5^2 \times 2^{27} \times 5^{27}$

$2^{27} \times 3 \times 5^{29}$ **(3)**

c) It is estimated that there are about 3.15×10^{79} protons in the universe.

Write this number as a product of its prime factors.

315×10^{77}

$3^2 \times 5 \times 7 \times 2^{77} \times 5^{77}$

$2^{77} \times 3^2 \times 5^{78} \times 7$ **(3)**

Prime Factors Problems

9) Consider the number $N = 3^4 \times 5^6 \times 13^8$

a) Fred multiplies N by a number to make it even.

i) What is the smallest number Fred could have chosen?
Write your answer as an integer.

2

ii) Explain your answer.

The number N is odd. You can make it even by doubling it.

(2)

b) Milly multiplies N by a number to make it a multiple of 42.

i) What is the smallest number Milly could have chosen?
Write your answer as an integer.

14

ii) Explain your answer.

Multiples of 42 must have prime factors of atleast $2 \times 3 \times 7$.
Hence we need 2×7.

(2)

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Question 9 Continued

$N = 3^4 \times 5^6 \times 13^8$

c) Ahmed multiplies N by a number to make it a square number.

i) What is the smallest number Ahmed could have chosen?
Write your answer as an integer.

$N = (3^2 \times 5^3 \times 13^4)^2$

1

ii) Explain your answer.

It is already square because of prime powers are even.

(2)

d) Abby multiplies N by a number to make it a cube number.

i) What is the smallest number Abby could have chosen?
Write your answer as an integer.

$3^2 \times 13$

117

ii) Explain your answer.

We need to make all the powers into multiples of 3.

(2)